GENERAL

hard probability questions with solutions

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Hard Probability Questions with Solutions: A Comprehensive Guide

Probability is a fascinating branch of mathematics that deals with the likelihood of events occurring. While many introductory problems are straightforward, real-world applications and advanced coursework often present challenges that require deeper insight, clever manipulation of formulas, and a solid grasp of underlying concepts. This article dives into a curated selection of hard probability questions, each accompanied by detailed step by step solutions. Whether you're a student preparing for competitive exams, a data analyst looking to sharpen your skills, or simply a curious mind, you'll find a wealth of practice problems that push the boundaries of conventional thinking.

Why Hard Probability Questions Matter

Mastering difficult probability problems does more than boost test scores; it develops a problem solving mindset that is applicable across disciplines:

- Analytical Thinking: Complex scenarios force you to decompose problems into manageable components.
- Pattern Recognition: Identifying underlying structures such as symmetry or independence becomes second nature.
- Mathematical Rigor: Advanced problems demand precise reasoning, reducing the likelihood of errors.
- Real World Relevance: Many engineering, finance, and scientific challenges involve probability at a sophisticated level.

Foundational Concepts Recap

Before tackling the hardest questions, let's briefly revisit key probability principles that will appear repeatedly throughout the solutions.

1. Basic Probability Rules

For any event *A* in a sample space *S*,

- Probability of A is P(A) = Number of outcomes in A / Number of outcomes in S.
- Complement rule: P(Ac) = 1 - P(A).
- Union rule: P(A "* B) = P(A) + P(B) - P(A ") B).

2. Conditional Probability & Independence

Given events *A* and *B*, the conditional probability of *A* given *B* is

- P(A|B) = P(A ") B) / P(B) (provided P(B) > 0).
- Events are independent if P(A ") B) = P(A)P(B).

3. Bayes' Theorem

Bayes' theorem connects conditional probabilities in reverse:

- P(A|B) = [P(B|A) * P(A)] / P(B).

4. Combinatorics Basics

- Permutations: nP_k = n! / (n-k)!

- Combinations: nC_k = n! / [k!(n-k)!]
- Multinomial coefficients for partitioning into more than two groups.

5. Random Variables & Expectation

A random variable *X* maps outcomes to real numbers. Its expectation is

- E[X] = :2 , ¢ …, Ò ,’ `or discrete X.
- Variance: Var(X) = E[(X - E[X])²] = E[X²] - (E[X])².

Hard Probability Questions – Categories

The following sections present problems grouped by theme, each illustrating a different facet of probability theory. Every question is followed by an exhaustive solution that explains the logic, calculations, and any subtle pitfalls.

1. Combinatorial Probability

These problems involve counting techniques combined with probability calculations. They often require careful handling of overlapping cases or constraints.

2. Conditional & Bayesian Inference

Here we explore scenarios where information updates the likelihood of events, demanding application of Bayes’ theorem and an understanding of dependencies.

3. Random Variables & Distributions

Questions that involve discrete or continuous distributions, expectations, variances, or moment generating functions.

4. Markov Chains & Stochastic Processes

These problems introduce state transitions over time, requiring matrix algebra and steady state analysis.

5. Advanced Topics (Poisson, Negative Binomial, etc.)

Problems that go beyond basic distributions, often involving limit theorems or approximations.

Problem Set 1: Combinatorial Probability

Question 1: Card Hand with a Twist

In a standard 52 card deck, a random 5 card hand is dealt. What is the probability that the hand contains exactly two aces and the remaining three cards are all from the same suit (but not aces)?

Solution

1. Count the favorable hands. We need:

   Choose 2 aces out of the 4 available: 4C2 = 6.
2. Choose a suit for the other three cards: 4 choices.
3. From the chosen suit, exclude the ace of that suit (if it was not already chosen). Since we have already chosen 2 aces, the ace of the chosen suit may or may not be among them. However, the problem specifies that the remaining three cards are not aces, so we must exclude the ace of that suit from the pool.
4. Number of non ace cards in a suit: 12 (13 total minus the ace).
5. Choose 3 cards from those 12: 12C3 = 220.

Key insight: Carefully separate the selection of aces from the suit selection for the remaining cards, ensuring no

accidental inclusion of an ace in the non ace group.

Question 2: Permutation Puzzle

How many permutations of the word “PROBABILITY” contain the substring “BIL”?

Solution

1. Identify repeated letters. The word has 11 letters with repeats: B appears 2 times, I appears 2 times, and all other letters are distinct.
2. Treat “BIL” as a single block. The block is fixed in order. The remaining letters to arrange are: P, R, O, A, B, I, T, Y, plus the block “BIL”. Count of items: 9.
3. Account for internal repeats. Within the block “BIL”, B and I are unique relative to the block, so no repeats inside the block. However, we still have an extra B and I among the remaining letters, each appearing once.
4. Number of distinct permutations of the 9 items. Since there are no repeats among the 9 items (the repeated letters are now part of the block), we have 9! = 362,880 arrangements.
5. Check for overcounting due to identical B and I. The block contains one B and one I. The other B and I are separate. Since the block is treated as a distinct entity, no further division is necessary.
6. Answer. There are 362,880 permutations containing the substring “BIL”.

Notice that the presence of the substring reduces the effective alphabet size, simplifying the counting process.

Problem Set 2: Conditional & Bayesian Inference

Question 3: Diagnostic Test Dilemma

A disease affects 1% of a population. A diagnostic test has a false positive rate of 5% and a false negative rate of 2%. If a patient tests positive, what is the probability that they actually have the disease?

Solution

1. Define events.

D = patient has disease.

2. T+ = test positive.

- P(D) = 0.01.
- P(T+|D) = 1 - false negative = 0.98.
- P(T+|Dc) = false positive = 0.05.

- P(D|T+) = [P(T+|D) * P(D)] / [P(T+|D) * P(D) + P(T+|Dc) * P(Dc)]
- Compute numerator: 0.98 × 0.01 = 0.0098.
- Compute denominator: 0.0098 + 0.05 × 0.99 = 0.0098 + 0.0495 = 0.0593.
- Result: 0.0098 / 0.0593 "H 0.1654.

Question 4: Monty Hall Variant

Suppose a game show has 4 doors: behind one door is a car, behind the others are goats. You pick a door. The host, who knows what’s behind each door, opens one of the remaining doors that contains a goat. You are offered a chance to switch to one of the two unopened doors. What is the probability of winning the car if you switch?

Solution

1. Initial choice. Probability you initially pick the car: 1/4. Probability you pick a goat: 3/4.
2. Host’s action. The host always opens a goat door among the remaining three. If you initially picked a goat, there are two goat doors left; the host chooses one uniformly at random.

3. After host opens a goat, remaining doors:

If your initial pick was the car, the two remaining unopened doors both contain goats.

4. If your initial pick was a goat, the remaining unopened doors are: one goat (the one not opened) and the car.

- Case 1 (initial car): 0 (since both remaining are goats).
- Case 2 (initial goat): 1/2 (since one of the two is the car).

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