GENERAL

stochastic calculus for finance 1 solutions

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Stochastic Calculus for Finance 1 Solutions: A Comprehensive Guide

When students first encounter the world of stochastic calculus in finance, the concepts can feel intimidating and abstract. The equations, the notation, and the underlying probability theory can create a steep learning curve. However, mastering these tools is essential for anyone aiming to excel in quantitative finance, risk management, or financial engineering. This article offers a detailed, step by step walkthrough of common problems found in the first course on stochastic calculus for finance, complete with clear explanations, worked solutions, and practical insights that bridge theory to real world applications.

Why Stochastic Calculus Matters in Finance

Financial markets are driven by uncertainty. Traditional deterministic models fall short when predicting asset prices, interest rates, or derivative payoffs. Stochastic calculus provides the mathematical language to model randomness precisely, allowing analysts to:

- Price complex derivatives such as options, futures, and swaps.
- Measure and manage risk using Value at Risk (VaR) and other metrics.
- Develop hedging strategies that reduce exposure to market movements.
- Model interest rates and credit spreads with dynamic, continuous time frameworks.

In the first course, students learn the foundations—Brownian motion, Ito's lemma, stochastic differential equations (SDEs), and the Black–Scholes framework. The solutions we present here will help you internalize these concepts and apply them confidently.

Key Concepts Covered in Stochastic Calculus for Finance 1

1. Brownian Motion and Wiener Processes

Brownian motion W(t) is a continuous time stochastic process with independent, normally distributed increments. It serves as the building block for all stochastic models in finance.

2. Ito's Lemma

Itô's lemma generalizes the chain rule to stochastic processes, enabling the transformation of functions of Brownian motion into differential equations. The formula is:

df = (" f/" t + 0.5<;" +&bò '‹"–GB ² Ã" f/" x dW(t)

3. Stochastic Differential Equations (SDEs)

Equations of the form dX(t) = ;Â…,ÇB–G@ + <2…,ÇB–Er‡B• describe the dynamics of stochastic processes. Solving SDE yields explicit expressions for asset prices, interest rates, or any other financial variable.

4. The Black–Scholes Model

By modeling the underlying asset as a geometric Brownian motion and applying Ito's lemma, the Black–Scholes partial differential equation (PDE) emerges. Its solution gives the closed form price of European options.

5. Risk Neutral Valuation

Under the risk neutral measure, the expected return of a tradable asset equals the risk free rate. This principle simplifies pricing by discounting expected payoffs at the risk free rate.

Common Problem Types in Stochastic Calculus for Finance 1

Below we categorize typical exercises and provide detailed solutions. Each problem is followed by a step by step walkthrough, ensuring clarity and reinforcing the underlying concepts.

Problem 1: Derive the Solution to a Geometric Brownian Motion

Given the SDE:

dS(t) = ;Å2‡B–GB ² ÃS(t)dW(t)

Find the explicit solution for S(t) with initial condition S(0) = S €.

1. Apply Ito's Lemma to ln S(t). Let f(S) = ln S. Then " f/" S = 1/S and " ²f/" S² = -1/S².
2. Compute df. Using Ito's lemma:

d(ln S) = (1/S)dS - 0.5(1/S²)(dS)²

Substitute dS from the SDE and note that (dW)² = dt:

d(ln S) = (1/S)(;Å6GB ² ÃS dW) - 0.5(1/S²)(<;%;" GB•

Simplify:

d(ln S) = (;Â Ò ãSÃ²)dt + <2 Er‡B•

"+ € W d(ln S) = (;Â Ò ãSÃ²)"+ € W dt + <2"² Õr Er‡B•

This yields:

ln S(t) - ln S € = (;Â Ò ãSÃ²)t + <5r‡B•

S(t) = S € exp[(;Â Ò ãSÃ²)t + <5r‡B•Ð

This is the celebrated solution for geometric Brownian motion.

Problem 2: Pricing a European Call Option Using Black–Scholes

Consider a non dividend paying stock with price S(t) following geometric Brownian motion. The risk free rate is r, volatility is <2Â æB F†R ÷ F–öâ Pxpires at time T with strike K. Derive the Black–Scholes price C(S,t).

1. Set up the risk neutral dynamics. Under risk neutral measure, the drift ;Â &V6öÖW2 .
2. Apply Ito's lemma to the option price V(S,t). The PDE is:

" V/" t + 0.5<;%;" +%bò %;" ² %2 %bò %2 Ò %b Ò

V(S,T) = max(S - K, 0) = (S - K)^+

<B Ò ãSÃ²(T - t),\\quad x = ln(S/K) + (r - 0.5<;"'…B Ò B•

C(S,t) = S N(d •) - K e^{-r(T - t)} N(d ,)

where

d • = [ln(S/K) + (r + 0.5<;"'…B Ò B•Ò ò ƒÃ" (T - t)),

d , = d • - <2!¢…B Ò B’à

Thus, the closed form solution is obtained.

Problem 3: Derive the Vasicek Interest Rate Model

Given the short rate SDE:

dr(t) = a(b - r(t))dt + <2 Er‡B•

Find the expectation E[r(t)] and variance Var[r(t)].

1. Solve the linear SDE analytically. This is a mean reverting Ornstein–Uhlenbeck process. The solution is:

r(t) = r € e^{-at} + b(1 - e^{-at}) + <2 Uç²Ö GÒ"² Õr Uç¶ 7Ò Er‡2•

E[r(t)] = r € e^{-at} + b(1 - e^{-at})

Var[r(t)] = <;" Uç²Ó& GÒ "² Õr Uç³& 7Ò G2 Ò Ã² (1 - e^{-2at}) / (2a)

These expressions provide the mean and spread of the short rate over time.

Problem 4: Verify Ito’s Lemma for a Quadratic Function

Let f(S) = S². Using Ito’s lemma, find df for S following geometric Brownian motion.

1. Compute partial derivatives. " f/" S = 2S, " ²f/" S² = 2.
2. Apply Ito’s lemma.

df = (" f/" t + 0.5<;%;" +&bò %;"–GB ² &bò %2 ÃS dW(t)

Since f has no explicit time dependence, " f/" t = 0. Thus:

df = (0.5<;%;" ¢ "–GB ² %2 ¢ ÃS dW(t) = <;%;" GB ² #ÃS² dW(t)

Hence, df = <;%;" GB ² #ÃS² dW(t).

Problem 5: Pricing a Zero Coupon Bond in the Hull–White Model

The Hull–White short rate model is:

dr(t) = [;,‡B’ Ò "‡B•ÖGB ² Ã dW(t)

Derive the bond price P(t,T) as a function of r(t).

1. Assume an affine form. Let P(t,T) = A(t,T) exp(-B(t,T) r(t)).
2. Derive differential equations for A and B. Substituting into the risk neutral pricing PDE yields:

dB/dt = a B(t,T) - 1,\\quad B(T,T) = 0

dA/dt = -;,‡B’ "‡BÅB’ ² ãSÃ² B(t,T)²,\\quad A(T,T) = 0

B(t,T) = (1 - e^{-a(T - t)}) / a

A(t,T) = exp\\Big[\\int_t^T (;,‡R’ "‡RÅB’ Ò ãSÃ² B(u,T)²) du \\Big]

P(t,T) = A(t,T) exp(-B(t,T) r(t))

Explicit expressions for A and B depend on the chosen ;,‡B’Â ut the affine structure remains.

Practical Tips for Tackling Stochastic Calculus Problems

• Keep track of the measure. Always specify whether you’re working under the physical or risk neutral measure; this affects the drift term.

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